Computer Science 336
Spring 2013
Homework 2a

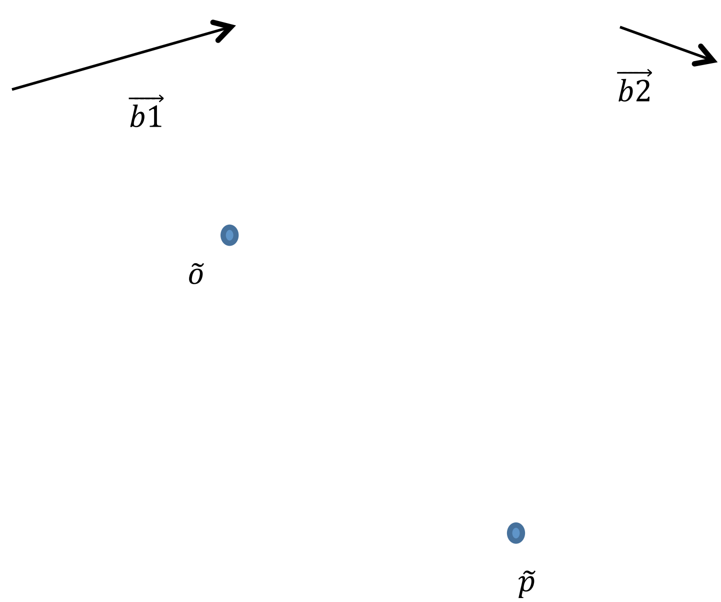
- Use the following notation for standard 3D affine transformation matrices. You can refer to these by name.

$$\texttt{RotateZ}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(Rotate about z-axis, i.e., in the x-y plane)}$$

$$\texttt{Translate(tx, ty, tz)} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\texttt{Scale(sx, sy, sz)} = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. Determine (geometrically) the approximate coordinates of the point $\tilde{p}$ below with respect to the frame $\mathcal{F} = [\overrightarrow{b1}, \overrightarrow{b2}, \tilde{o}]$. Justify how you got your answer using a sketch and/or brief explanation. Make a careful sketch and try to estimate the numbers to (say) one decimal place.

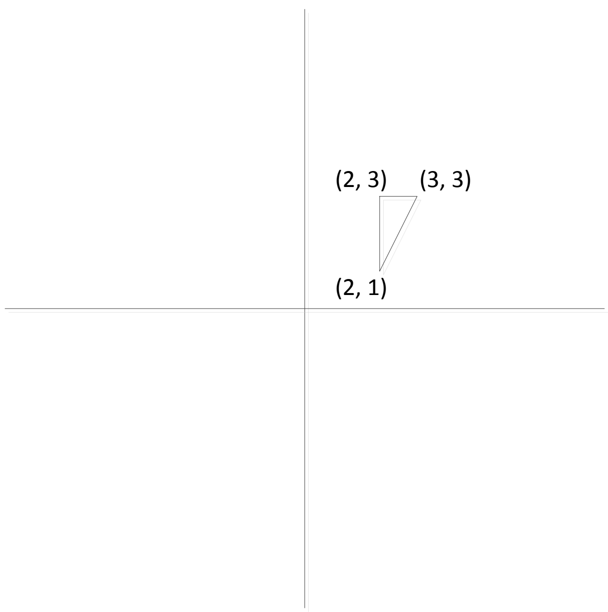


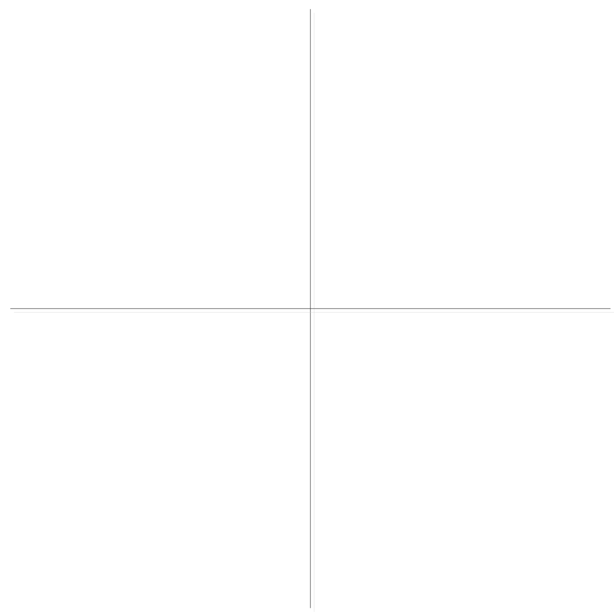
2.

$$\texttt{A = Translate(1,-3,0)} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \texttt{B = Rotate(90)} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

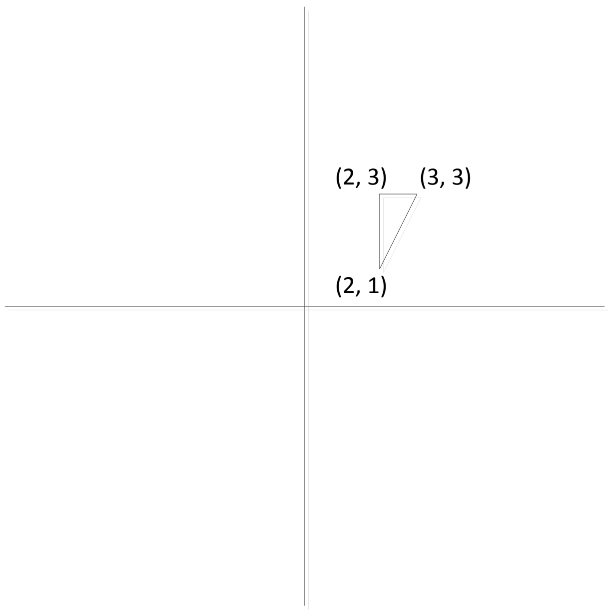
a) Find the two products AB and BA.

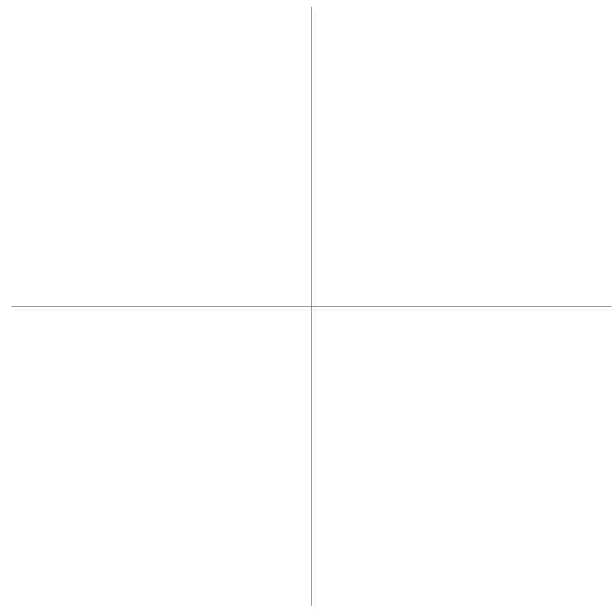
b) In the axes on the right, sketch the result applying the transformation BA (A, then B) to the triangle. Use the matrix BA to calculate the new coordinates of the three vertices, label them on your sketch, and make sure your result makes sense geometrically. Remember that a 2D point such as (2, 3) is represented in homogeneous coordinates as $[2\ 3\ 0\ 1]^T$.





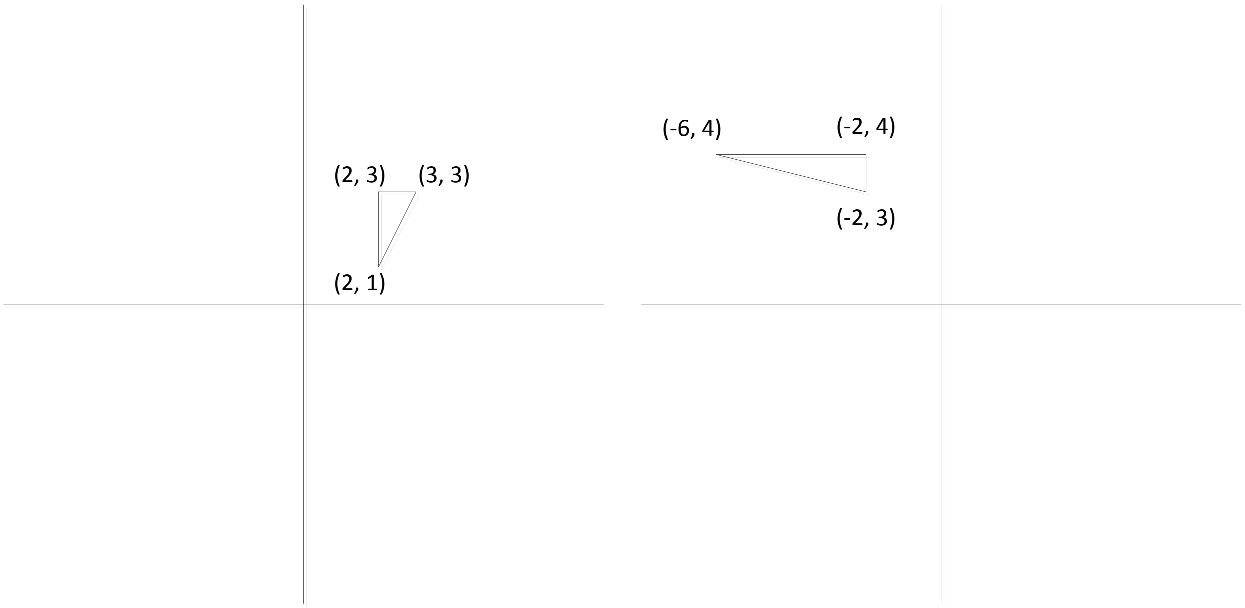
c) Do the same for the transformation AB.



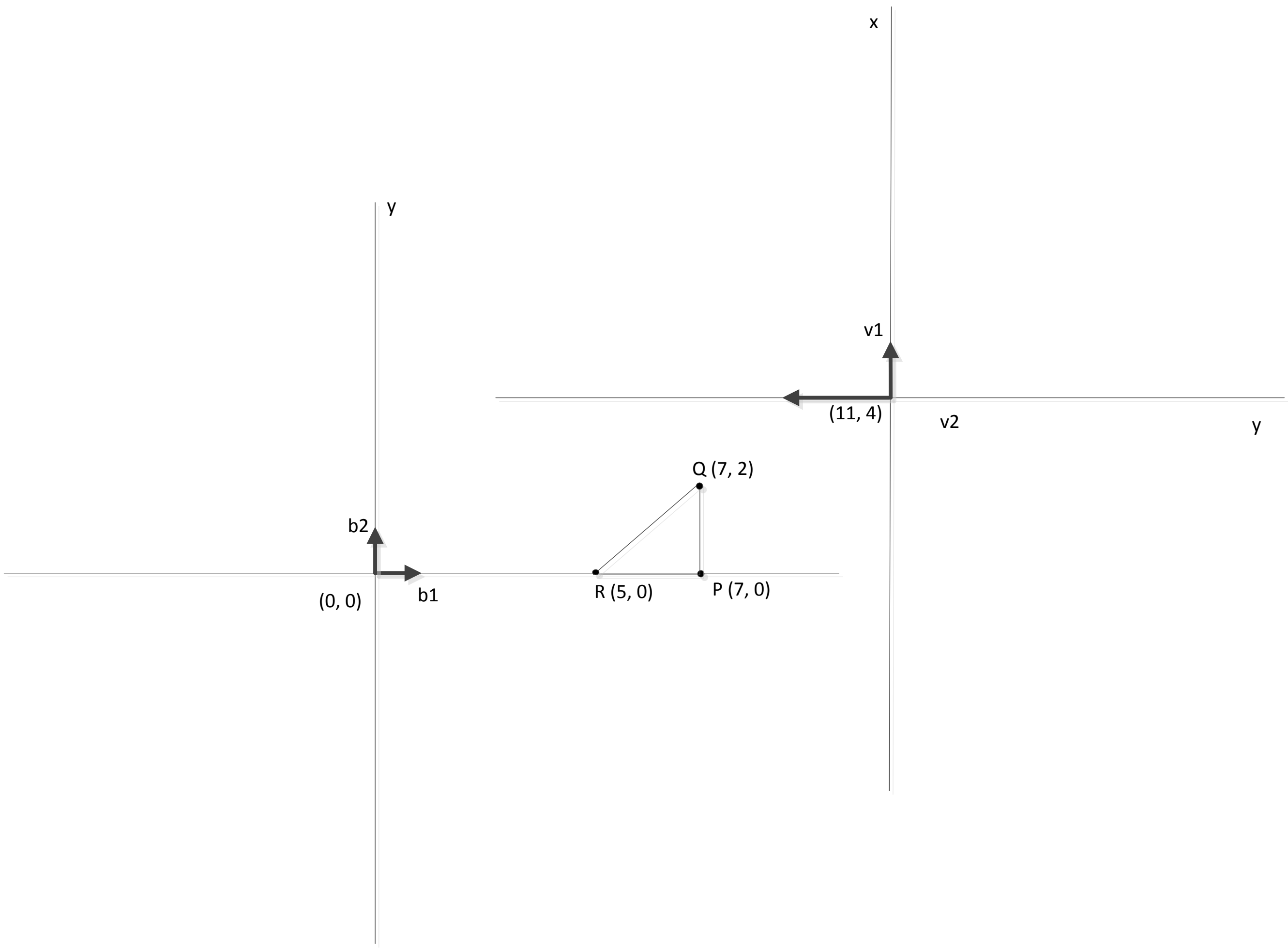


c) A and B are affine transformation matrices. Any product of affine transformations is also affine. Therefore BA (given to us as a translation followed by a rotation) is affine. An affine transformation is defined as a linear transformation followed by a translation. So there must be a way to write BA as a product of matrices TM such that M is linear and T is a translation. Find T and M. (A 4x4 matrix represents a *linear* transformation if the bottom row is [0 0 0 1] and the rightmost column is $[0\ 0\ 0\ 1]^T$.)

3. A certain affine transformation transforms the triangle on the left into the triangle on the right. Write down a matrix for this transformation as a product of standard transformation matrices. (There are many possible correct answers.)



4. The figure below shows a frame (coordinate system) $\mathcal{F} = [\overrightarrow{b1}, \overrightarrow{b2}, \overrightarrow{b3}, \tilde{o}]$ along with a second frame $\mathcal{F}' = [\overrightarrow{v1}, \overrightarrow{v2}, \overrightarrow{v3}, \tilde{p}]$, where the point $\tilde{p}$ has coordinates $[11\ 4\ 0\ 1]^T$ with respect to $\mathcal{F}$. The new frame is rotated 90 degrees counterclockwise and the y-axis is stretched by 2. Note that v1 and v2 are orthogonal, v1 has unit length, and v2 has length 2. (You can ignore the invisible third basis vectors $\overrightarrow{b3}$ and $\overrightarrow{v3}$ since nothing is changing outside of the x-y plane.) Thus the matrix M = Translate(11, 4, 0) * RotateZ(90) * Scale(1, 2, 1) transforms $\mathcal{F}$ to $\mathcal{F}'$, i.e. $\mathcal{F}' = \mathcal{F}M$.



a) When the triangle PQR is transformed into the new frame, where does it end up? Sketch it and **label** the coordinates with respect to the original frame $\mathcal{F}$, for example, the point R would end up at (11, 9). Use the matrix M to calculate the coordinates of the transformed triangle with respect to the original frame. Show your calculations and verify geometrically that your results make sense (the drawing is to scale).

b) Determine (geometrically) the coordinates, *with respect to* $\mathcal{F}'$, of the points labeled P, Q, and R. Then, find the inverse M^{-1} of the matrix M from (b). Calculate M^{-1}c for the three coordinate vectors for P, Q, and R and show that your answers agree with the geometrically obvious ones.